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COMMENT

Comment on 'Random sequential packing in square cellular structures'

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Abstract. We correlate two methods for packing squares on a square lattice and find agreement with published computer simulation results.

In a recent paper Nakamura (1986) has studied the problem of random sequential packing of squares on a square lattice using a computer simulation method. Squares with integer length a were inserted at random one by one without overlap onto a square lattice of size n constructed from unit cells. The packing fraction ϕ is the fractional area coverage of a configuration of squares to which it is impossible to add further squares.

Nakamura has distinguished two methods of placing the squares and deduced discrepancies in the transition to the corresponding continuum problem. However, we note that the two methods (A, in which contact between packed squares is allowed and, B, in which it is forbidden) are not independent. Non-contacting squares of side a may be transformed to contacting squares of side $(a+1)$ by adding a border of one-half unit and making a relative displacement of pattern and lattice of $(\frac{1}{2}, \frac{1}{2})$. With the obvious correction for evaluating ϕ and neglecting edge effects we obtain

$$\phi^{A}_{a+1} = (a + 1/a)^2 \phi^{B}_{a}. \qquad (1)$$

The validity of (1) is established, for example, from the first six entries of table 1 of Nakamura (see table 1).

Also from (1), in the limit of large squares ($n \to \infty$, $1/a \to 0$), packing fractions ϕ^{A} and ϕ^{B} must approach the same value. This is in disagreement with the two distinct limits obtained by Nakamura's extrapolation method. The unique fraction is the 'random parking fraction' of oriented squares (Finegold and Donnell 1979).

Table 1. Comparison of random parking fractions.

	a					
	1	2	3	4	5	6
ϕ^{B}_{a}	0.187	0.302	0.364	0.404	0.429	0.447
$(a+1/a)^2\phi^{B}_{a}$	0.748	0.680	0.647	0.631	0.618	0.608
ϕ^{A}_{a}	1.000	0.749	0.681	0.646	0.628	0.620

The boundary conditions imposed by Nakamura are cellular (not periodic) leading to an apparent distinction between ϕ^A and ϕ^B for finite (small) n which results from the asymmetry of B packing with respect to box walls or other internal squares. (Squares may touch the walls in method B.) Using the previous border construction it can be seen that method B used in a finite grid of size n corresponds to an effective grid size $(n+1)$. Equation (1) now holds for finite-size grids subject to the replacement

$$\phi_a^B \rightarrow (n/n+1)^2 \phi_a^B. \tag{2}$$

Again, data taken directly from Nakamura's figure 4 confirm this correction to (1) (e.g., $n=20$; $(a/n, \phi_2^A) = (0.10, 0.73)$ and $(a/n, \phi_1^B) = (0.05, 0.20)$).

These results indicate only one method of parking squares on a two-dimensional square lattice. Finally we note that equation (1) is a particular feature of the parking of symmetric shapes (i.e. shapes whose geometry is unaffected by addition of a border) and methods A and B will lead to distinct results for parking of irregular shapes (Barker and Grimson 1987).

References

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